1. [7 marks] Go to the course website, find the data section, and download the 'Lake Huron' dataset. Import this dataset into either `R` or `MATLAB`, and plot it with suitable axis labels and any other adjustments you feel are suitable. Fit a linear least-squares regression line to the data, and plot it over top of the original data (be sure to use a different colour). Compute ∇X_t where X_t is the lake level data, and plot this as well.

2. [6 marks] Let $\{Z_t\}$ be IID $\mathcal{N}(0,1)$ noise, and define

$$X_t = \begin{cases} Z_t, & \text{if } t \text{ is even,} \\ (Z_{t-1}^2 - 1)/\sqrt{2}, & \text{if } t \text{ is odd.} \end{cases}$$

(a) Show that $\{X_t\}$ is WN$(0,1)$ but not IID$(0,1)$ noise.

(b) Find $\mathbf{E}\left[X_{n+1} \middle| X_1, \ldots, X_n\right]$ for n odd and n even, and compare the results.

(this is problem 1.8 in your text)

3. [8 marks] Suppose $\{a_t\}$ are IID RVs with mean 0 and variance σ^2, and define a stochastic process $\{Y_t\}$ by $Y_t = \mu + a_t + \frac{3}{4}a_{t-1}$, $t = \ldots, -1, 0, 1, 2, \ldots$ where μ is a constant. Find the mean, autocovariance function, and ACF of $\{Y_t\}$, and verify that $\{Y_t\}$ is a stationary process. Also find the ACF of the process defined by $Y_t = \mu + e_t + \frac{4}{3}e_{t-1}$, where the e_t are IID with mean 0 and variance σ_e^2, and compare and comment on the ACFs of the two processes.

4. [5 marks] Show that a linear filter $\{a_j\}$ passes an arbitrary polynomial of degree k without distortion, i.e. that

$$m_t = \sum_j a_j m_{t-j}$$

for all k^{th} degree polynomials $m_t = c_0 + \sum_{i=1}^k c_i t^i$ iff

$$\begin{cases} \displaystyle\sum_j a_j = 1 & \text{and} \\ \displaystyle\sum_j j^r a_j = 0 & \text{for } r = 1, \ldots, k. \end{cases}$$

(this is problem 1.12a in your text)

5. [4 marks] Review the Spencer 15-point moving average filter $\{a_j\}$ (page 27, text), and show that it does not distort a cubic trend, i.e. that if $m_t = c_0 + c_1 t + c_2 t^2 + c_3 t^3$, then $\displaystyle\sum_{i=-7}^{7} a_i m_{t+i} = m_t$.

(for graduate students, or undergraduates seeking extra credit)

6. [5 marks] Implement a function in your program of choice that takes a time series as input, and returns the first lag$-k$ autocovariance coefficients. Include an option to return autocorrelation coefficients as well. Hand in the function as a neatly formatted printout. Comment where necessary. *Preference is given for programs written in R or MATLAB.*

7. [7 marks] For the AR(1) process

$$x_t = \alpha x_{t-1} + \epsilon_t$$

where $\{\epsilon_t\}$ is a sequence of independent, zero-mean, Gaussian random variables, and $\alpha \in \mathbb{R}$, $\alpha \in (-1, 1)$, find:

1. The process variance $\gamma_0 = \sigma_x^2 = \mathbf{E}\left[x_t^2\right]$
2. The lag-2 autocovariance $\gamma_2 = \mathbf{E}\left[x_t x_{t+2}\right]$
3. The lag-k autocorrelation $\rho_k = \dfrac{\mathbf{E}\left[x_t x_{t+k}\right]}{\sigma_x^2}$

8. [4 marks] If $m_t = \sum_{k=0}^{p} c_k t^k$, $t = 0, \pm 1, \ldots$, show that ∇m_t is a polynomial of degree $(p-1)$ in t, and hence that $\nabla^{p+1} m_t = 0$.