DEPARTMENT OF MATHEMATICS AND STATISTICS

QUEEN'S UNIVERSITY AT KINGSTON

APSC 171 - DECEMBER EXAM 2011 - VERSION 1

INSTRUCTIONS:

- Answer **all questions**, writing clearly in the space provided. If you need more room, continue your answer on the back of the **previous page**, providing clear directions to the marker.
- **Show** all your work and **explain** how you arrived at your answers, unless explicitly told to do otherwise.
- Only **CASIO FX-991**, **Gold Sticker** calculators are permitted.
- Write your student number **clearly** at the top of **each** page.
- You have three hours to complete the examination.
- Wherever appropriate, **include units in your answers.**
- When drawing graphs, **add labels and scales on all axes.**

PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

Problem	Possible	Received
1,2	10	
3,4	11	
5	9	
6	7	
7	7	
8	8	
9	9	
10	8	
11	11	
TOTAL	80	

1. Solve for $\dfrac{d\vec{s}}{dt}$ given that [5 marks]

$$\vec{s}(t) = \langle \ \sin(\tan(t)), \quad t^3 \cdot 2^t, \quad [\ln(t)]^t \ \rangle$$

2. Evaluate $\displaystyle\int_1^8 \frac{1}{\sqrt[3]{x}} \ \sin(\pi x^{2/3}) \ dx$. [5 marks]

3. Solve for $\frac{dy}{dx}$ given that $x^2y^2 + \arctan(x) \cdot y = 10$. [3 marks]

4. Determine whether the integral $\int_0^\infty \frac{x-1}{(x+1)^2}\, dx$ converges or diverges. If it converges, find its value. [6 marks]

Reminder from the front page: you must show all your work and explain how you arrived at your answers for all questions.

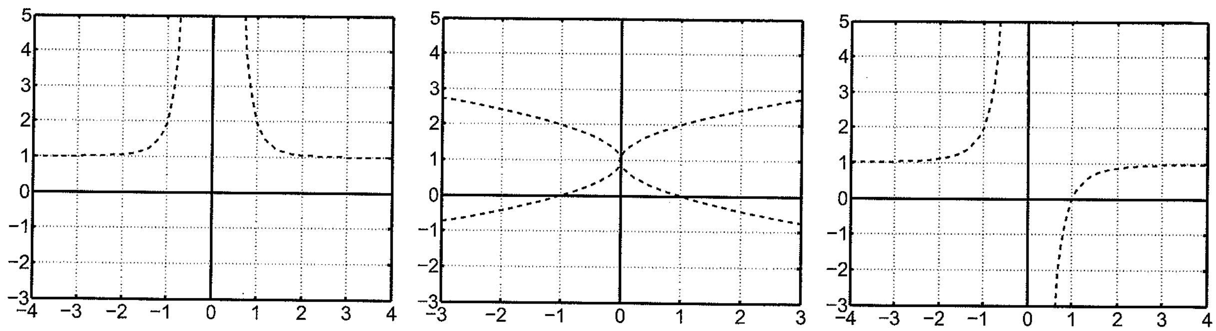
5. Consider a particle whose (x, y) position is given by $\vec{s}(t) = \langle \frac{-1}{\sqrt{t}},\ t^2 + 1 \rangle$. Both x and y are in meters and t is in seconds.

   (a) Find the speed of the particle at $t = 1$. [2 marks]

   (b) Find the equation of the (x, y) graph of the particle's path by eliminating the variable t in $\vec{s}(t)$. [4 marks]

(c) The shape of the (x, y) graph of $\vec{s}(t)$ is shown on one of the sets of axes below. [2 marks]

- Circle the correct axes.
- Fill in the subset of the graph traversed by the particle as time increases, **starting at** $t = 1$. Indicate the direction followed by the particle with an arrow on the graph.



(d) Is the particle speeding up or slowing down at the instant $t = 1$? [3 marks]

6. A compressed dry sponge is initially 20 cm in length, and 5 cm in both width and height. It is dropped in water and begins to expand. You note that

- the length increases at a rate of 3 cm/min,
- the volume increases at a rate of 120 cm^3/min, and
- the width and height remain equal to each other.

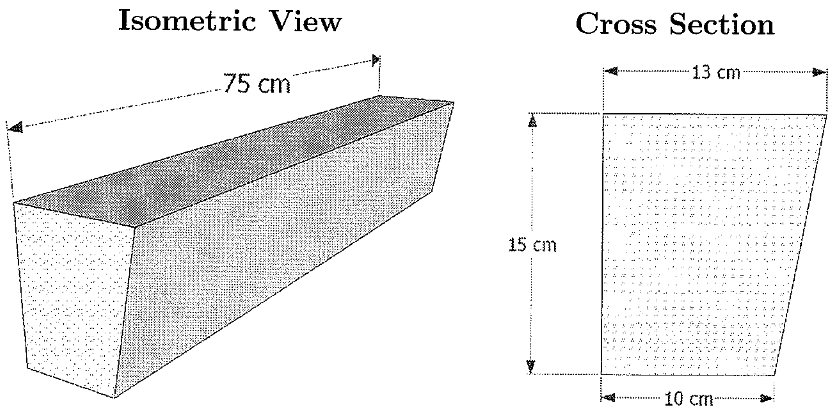
Reminder from the front page: include units in all answers when they are available.

(a) At the moment the sponge is dropped in the water, how fast is its width increasing? [5 marks]

(b) At what rate is the surface area of the sponge increasing at this same moment? [2 marks]

7. The finite region bounded by the curve $y = 1 - x^2$ and the line $y = 0$ is rotated around the line $x = 1$. Sketch the 2D region to be rotated, and find the volume of revolution. [7 marks]

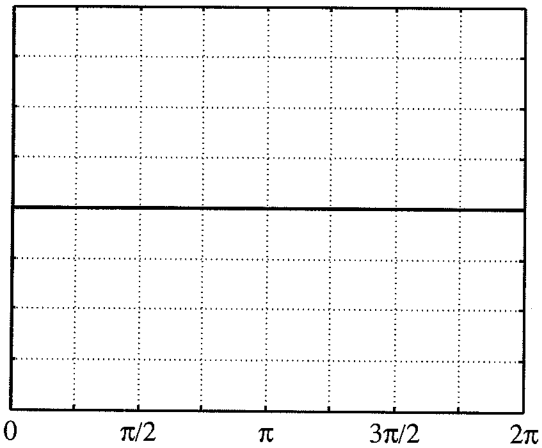
8. A box for holding plants is attached to the outside of a house. It is 75 cm long, and has a trapezoidal cross-section, with the dimensions indicated on the diagrams below. [8 marks]



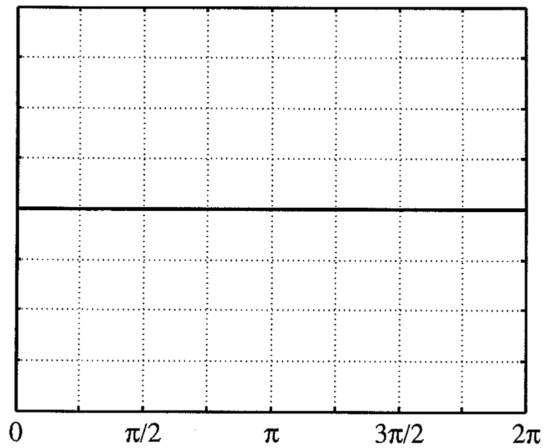
During a rainstorm, this box fills up to the top with water. Given the dimensions of the box, and the fact that the bottom of the box is 1.5 m above the ground, compute the total gravitational potential energy of the water in the box, relative to the ground. Reminder: the density of water is 1000 kg/m^3.

For any variables you introduce, define them either in a sentence or on the diagram.

9. (a) Sketch the graph of $f(x) = x\sin(x)$ on the **first** set of axes below. Clearly indicate the vertical scale you use. [1.5 marks]



Graph of $f(x) = x\sin(x)$



Trapezoidal Approx. to $\int_0^{2\pi} x\sin(x)\,dx$

(b) On the **second** set of axes, sketch the shapes representing the **trapezoidal** estimate of the integral $\int_0^{2\pi} x\sin(x)\,dx$, using 4 intervals. [1.5 marks]

(c) Compute the value of the trapezoidal estimate with 4 intervals of the integral $\int_0^{2\pi} x\sin(x)\,dx$. [2 marks]

(d) Find the exact value of the integral $\int_0^{2\pi} x\sin(x)\,dx$ using anti-derivatives. [4 marks]

10. Consider the spring/mass system studied in class, with mass $m = 25$ kg and spring constant $k = 16$ N/m. The position of this mass over time will be determined by the differential equation

$$25\frac{d^2x}{dt^2} = -16x(t)$$

(a) Find one solution to this differential equation. Any function $x(t)$ that satisfies the differential equation will be marked correct. [1 mark]

(b) Find a second solution to this differential equation that also satisfies the initial conditions $x(0) = 3$ and $x'(0) = 4$. [4 marks]

(c) What is the period of the oscillations of the mass in this system? [1 mark]

(d) If the mass were to be doubled (i.e. the 25 kg replaced by 50 kg), by what factor would the period of the mass' oscillations change? Write your answer in the form of a sentence, and explain your reasoning. [2 marks]

11. Heparin, a blood-thinning drug, is an important treatment used to reduce blood clots, and its level must be very carefully monitored for safe use. We will use a differential equation model to study how the amount of heparin in a patient changes over time.

    (a) Let $H(t)$ be the amount of heparin in the body (in mg) at time t (hours). For this patient, the heparin is

    - injected continuously at a rate of 4 mg/hr, and
    - metabolized (broken down) continuously at a rate proportional to H.
        - The constant of proportionality is 0.8 (with units (1/hours)).

    Write the differential equation that represents the *net rate of change* of H. [2 marks]

    (b) Solve this differential equation to find an expression for $H(t)$, given that treatment starts at time $t = 0$, so $H(0) = 0$. [4 marks]

(c) Find the mass of heparin in the patient's body reached after a long period of treatment. (Note: this part can be solved using just the information in part (a), if you were unable to answer part (b).) [1 mark]

(d) Consider now a patient whose treatment with heparin is being halted, so that only metabolism is affecting their heparin levels.

- The patient currently has a level of 12 mg of heparin in their body, and
- a level of 0.5 mg is the lowest still considered to be clinically effective.

How long does it take after the drug supply is stopped for their heparin level to drop below the clinically effective threshold? [4 marks]

CORRECTION NOTICE

APSC 171 - Mon Dec 12 2011 - Minor error on the front page.
The mark distribution for questions 3,4 and 5 should be the following:

Problem	Possible	Received
1,2	10	
3,4	**9**	
5	**11**	
6	7	
⋮	⋮	
TOTAL	80	