Student Number ____________________

Instructor ____________________ Section ____________

QUEEN'S UNIVERSITY, FACULTY OF APPLIED SCIENCE
APSC 171 FINAL EXAMINATION, DECEMBER, 2009
D. AGUILAR, L. JONKER, M. ROTH

- The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretation of any question that requires a written answer.
- PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.
- Answer in the spaces provided on the question paper. If necessary, an answer may be continued on THE BACK OF THE PREVIOUS PAGE.
- You may use calculators with a GOLD sticker.
- SHOW HOW YOU REACH YOUR RESULTS. Marks are not given for a correct answer alone. State or display answers in an appropriate way.
- You may write in pencil, but write clearly. Do not write in red ink.
- Except where a decimal answer is asked for, it is preferable to leave answers in the form $\sqrt{\pi}$, e^2 and so on. However, do any obvious simplification (for example $2 + \frac{1}{2} + \frac{1}{3} = 2\frac{5}{6}$ or $\frac{17}{6}$, $\frac{(x+1)^2}{(x+1)} = (x+1)$).
- Marks per question or part question are shown in square brackets on the right margin (for example [4]). The total number of marks is 75.
- Check that your question paper has 10 pages.

FOR EXAMINER'S USE ONLY		
Page	Mark Available	Mark
2	8	
3	11	
4	9	
5	5	
6	6	
7	7	
8	9	
9	9	
10	11	
Total	75	

1. Calculate $\frac{dy}{dx}$ in each of the following cases:

(a) $y = \frac{\cos(x^2+1)}{\sin^2(x)+1}$ [3]

(b) $y = (\arctan x)^{x^3}$ [5]

2. Calculate $\int_1^4 \frac{\sqrt{x}+x^2+1}{3x\sqrt{x}}\,dx$ [6]

3. Calculate $\int_0^{\pi} \frac{\sin(x)}{1+\cos^2(x)}\,dx$ [5]

4. Suppose waste water from a manufacturing process flows into a holding tank at a rate of $f(t)$ kg per hour, and that at time t (measured in hours) the concentration of salt in the waste water is $s(t)$ ppm (parts per million); that is, it is at a concentration of $s(t)$ kg of salt per 1,000,000 kg of water. The following table records the values of $f(t)$ and $s(t)$ at various times:

t (hours)	0	2	4	6	8	10	12
$f(t)$ (kg)	1.5×10^2	1.7×10^2	1.6×10^2	1.4×10^2	1.2×10^2	0.9×10^2	1.2×10^2
$s(t)$ (ppm)	47	49	50	48	50	51	49

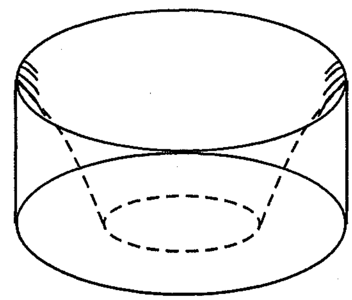
(a) If you had formulas for f and s, what integral would calculate the increase in the mass of salt in the holding tank between times $t = 0$ and $t = 12$? [4]

(b) Interpret $\frac{d}{dx}\int_1^x f(t)\,dt$ (describe what it means physically) and determine, from the information provided, its value at time $x = 4$. [2]

(c) Use Simpson's Rule to estimate the total amount of salt added to the holding tank between times $t = 0$ and $t = 12$. [3]

5. Consider the region D bounded by the x-axis, the line $x = e$ and the curve $y = \ln x$. If D is rotated around the y-axis it produces a solid E that looks like a cylinder with a trumpet-shaped hole through the middle. We assume that the units on the axes are meters, that this solid is made of material whose density is $\rho = 0.8$ kg per cubic meter, and that it is rotating around the y-axis with an angular velocity $\omega = 5$ radians per second. Recall from Physics that if m is the mass of a tiny piece of E, located at distance x from the axis of rotation, then the kinetic energy of this one tiny piece is given by $(1/2)m\omega^2x^2$ Joules. We want to use cylindrical shells to calculate the kinetic energy of the rotating solid E.

   (a) Explain why in this problem we should use cylindrical shells as opposed to horizontal slices to calculate the total kinetic energy of E. [1]



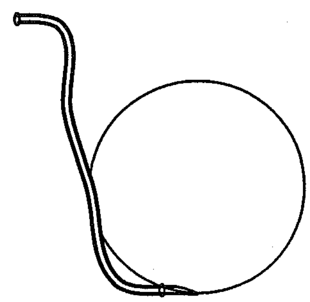
   (b) Find an expression for the kinetic energy of a single cylindrical shell of radius x and thickness dx. [4]

Question 5 continued...

(c) Calculate the kinetic energy of E. [6]

6. Find the volume of the solid produced when the region under the graph of $y = \dfrac{1}{x^2 + 6x + 5}$ and the lines $x = 0$ and $x = 1$ is rotated about the line $x = -3$. [7]

7. An underground spherical oil tank with two-meter radius is half full of oil. A hose is attached to a valve at the very bottom of the tank. The other end of the hose is five meters above the bottom of the oil tank, and is attached to a tanker truck. If the density of oil is 900 $\mathrm{kg/m^3}$, and the acceleration due to gravity is $g = 9.8\ \mathrm{m/s^2}$, what is the minimal amount of work required to pump the oil into the truck? [9]



8. Newton's Law of Cooling tells us that the rate at which a potato heats up in a hot oven is proportional to the difference between the temperature of the potato and the temperature of the oven.

   (a) Assuming that the oven is at 180° translate this into a mathematical sentence, clearly defining each of the variables and constants needed to do that, and introducing no more variables than necessary. [2]

   (b) If the initial temperature of the potato is 20°C, and if it takes 1 minute for the potato to heat up to 30°C, what will its temperature be at the end of 2 minutes? [7]

9. The paths of the two spacecraft whose positions at time t are given by $\mathbf{r}(t) = t^3\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}} + t\,\hat{\mathbf{k}}$ and $\mathbf{u}(t) = (t^2 - 2t)\,\hat{\mathbf{i}} + t\,\hat{\mathbf{j}} + (t-2)\,\hat{\mathbf{k}}$ cross at the point $(-1, 1, -1)$ (they do not collide, however). Time is measured in seconds and distance in meters.

   (a) What is the speed of the first spacecraft when it passes through $(-1, 1, -1)$? [3]

   (b) Do the trajectories cross at any other points? [4]

10. Calculate the improper integral $\displaystyle\int_{-\infty}^{0} \frac{1}{(8-x)^{7/3}}\,dx$. [4]