QUEEN'S UNIVERSITY
APSC 171J – Solutions to Assignment 1
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Due: January 17, 2013

INSTRUCTIONS

- This assignment is due in-class (4:30-5:20pm) Thursday, January 17
- Answer all questions, writing clearly on the sheets provided. **You must print this file and hand in a carefully stapled copy!** Unstapled assignments will not be accepted.
- One mark in each question is for **complete** (and mostly correct) work shown
- The second mark is for a **fully** correct solution, which **must** be placed in the box provided
- Whenever possible, simplify your solution.
- There are no part marks: you will receive 0, 1 or 2 on each question.
- Future assignments will contain more problems (and correspondingly more overall marks), but due to the timing of the first week of the compressed term, this assignment has only five problems on it.

FOR INSTRUCTOR'S USE ONLY		
Question	Mark Available	Received
1	2	
2	2	
3	2	
4	2	
5	2	
TOTAL	10	

1. [2 marks] Find $\frac{dy}{dx}$ for

$$y = [1 + 2xe^x]^4$$

using careful application of the Chain Rule, Product Rule and Power Rule.

Showing all steps:

$$\begin{aligned}\frac{d}{dx}(y) &= 4\left[1+2xe^x\right]^{4-1} \cdot \frac{d}{dx}\left[1+2xe^x\right] \text{ (Power Rule and Chain Rule)}\\ &= 4\left[1+2xe^x\right]^3 \cdot \left[\frac{d}{dx}(2x)\cdot e^x + (2x)\cdot\frac{d}{dx}e^x\right] \text{ (Product Rule)}\\ &= 4\left[1+2xe^x\right]^3 \cdot \left[2e^x + 2xe^x\right]\\ &= 8e^x\left(1+2xe^x\right)^3(1+x)\,.\end{aligned}$$

Final Answer:

$$y'(x) = 8e^x\left(1+2xe^x\right)^3(1+x)\,.$$

2. [2 marks] Find $\frac{dy}{dx}$ for

$$y = x \cdot \sin\left(e^{x^2+x}\right)$$

using careful application of the Chain Rule, Product Rule and Power Rule.

Again, showing all steps:

$$\begin{aligned}
\frac{d}{dx}(y) = y' &= \frac{d}{dx}(x) \cdot \left(\sin\left(e^{x^2+x}\right)\right) + (x) \cdot \frac{d}{dx}\left(\sin\left(e^{x^2+x}\right)\right) \quad \text{(Product Rule)} \\
&= (1) \cdot \left(\sin\left(e^{x^2+x}\right)\right) + (x) \cdot \left[\cos\left(e^{x^2+x}\right) \cdot \frac{d}{dx}\left(e^{x^2+x}\right)\right] \quad \text{(Chain Rule)} \\
&= \left(\sin\left(e^{x^2+x}\right)\right) + x\left(\cos\left(e^{x^2+x}\right)\right)\left(e^{x^2+x}\frac{d}{dx}\left(x^2+x\right)\right) \quad \text{(Chain Rule again)} \\
&= \left(\sin\left(e^{x^2+x}\right)\right) + x\left(\cos\left(e^{x^2+x}\right)\right)\left(e^{x^2+x}(2x+1)\right) \\
&= \sin\left(e^{x^2+x}\right) + (2x^2+x)e^{x^2+x}\cos\left(e^{x^2+x}\right).
\end{aligned}$$

Final Answer:

$$y'(x) = \sin\left(e^{x^2+x}\right) + (2x^2+x)e^{x^2+x}\cos\left(e^{x^2+x}\right).$$

3. [2 marks] Find $\frac{dy}{dx}$ for

$$y = \cos\left(\csc(x^2)\right)$$

remembering to apply the Chain Rule as many times as necessary.

Using the identity for $\csc(x)$:

$$\begin{aligned} y'(x) &= -\sin\left(\csc(x^2)\right)\cdot\frac{d}{dx}\left(\csc(x^2)\right) \text{ (Chain Rule)} \\ &= -\sin\left(\csc(x^2)\right)\cdot\frac{d}{dx}\left(\frac{1}{\sin(x^2)}\right) \text{ (using identity for csc)} \\ &= -\sin\left(\csc(x^2)\right)\cdot\left(\frac{0-(1)\cos(x^2)(2x)}{\sin^2(x^2)}\right) \text{ (Quotient and Chain Rules)} \\ &= \frac{2x\sin\left(\csc(x^2)\right)\cos(x^2)}{\sin^2(x^2)} \end{aligned}$$

Alternatively:

$$\begin{aligned} y'(x) &= -\sin\left(\csc(x^2)\right)\cdot\frac{d}{dx}\left(\csc(x^2)\right) \text{ (Chain Rule)} \\ &= -\sin\left(\csc(x^2)\right)\cdot\left[-\csc(x^2)\cot(x^2)\cdot\frac{d}{dx}\left(x^2\right)\right] \text{ (Chain Rule and Memorization)} \\ &= 2x\sin\left(\csc(x^2)\right)\csc(x^2)\cot(x^2) \\ &= \frac{2x\sin\left(\csc(x^2)\right)\cos(x^2)}{\sin(x^2)\sin(x^2)} \text{ (expanding)} \\ &= \frac{2x\sin\left(\csc(x^2)\right)\cos(x^2)}{\sin^2(x^2)} \end{aligned}$$

Final Answer:

$$y'(x) = \frac{2x\sin\left(\csc(x^2)\right)\cos(x^2)}{\sin^2(x^2)}$$

4. [2 marks] Find $\frac{dy}{dx}$ for

$$y = \ln\left(\frac{x^2-4}{2x+5}\right) \text{ for } x > 2.$$

The $x > 2$ was included so we didn't have to worry about negative inputs to the $\ln(x)$ function.

$$\begin{aligned}
y'(x) &= \frac{1}{\frac{x^2-4}{2x+5}} \cdot \frac{d}{dx}\left(\frac{x^2-4}{2x+5}\right) \text{ (Memorization and Chain Rule)} \\
&= \frac{2x+5}{x^2-4} \cdot \left[\frac{\frac{d}{dx}\left(x^2-4\right)\cdot(2x+5) - (x^2-4)\frac{d}{dx}(2x+5)}{(2x+5)^2}\right] \text{ (Quotient Rule)} \\
&= \frac{2x+5}{x^2-4} \cdot \frac{(2x)(2x+5)-(x^2-4)(2)}{(2x+5)^2} \\
&= \frac{2x+5}{(2x+5)(2x+5)(x^2-4)}\left(2x(2x+5)-(x^2-4)(2)\right) \\
&= \frac{4x^2+10x-2x^2+8}{(2x+5)(x^2-4)} \\
&= \frac{2x^2+10x+8}{(2x+5)(x^2-4)} \\
&= \frac{2(x+4)(x+1)}{(2x+5)(x-2)(x+2)}
\end{aligned}$$

Final Answer:

$$y'(x) = \frac{2(x+4)(x+1)}{(2x+5)(x-2)(x+2)}$$

5. [2 marks] Find $\frac{dy}{dx}$ for

$$y(x) = \cot(x) = \frac{\cos(x)}{\sin(x)}.$$

by using the Quotient Rule and the identity provided. Simplify your answer.

This question intended your simplification to be full (that is, to the derivative solution that you are expected to memorize). If you got the alternative (not "as correct") solution, you will still get full marks – I should have been more precise in the question.

$$\begin{aligned}
\frac{d}{dx}(y) &= \frac{d}{dx}\left[\frac{\cos(x)}{\sin(x)}\right] \\
&= \frac{\frac{d}{dx}(\cos(x)) \cdot \sin(x) - (\cos(x))\frac{d}{dx}(\sin(x))}{\sin^2(x)} \text{ (Quotient Rule)} \\
&= \frac{-\sin(x)\sin(x) - \cos(x)\cos(x)}{\sin^2(x)} \\
&= \frac{-\left(\sin^2(x) + \cos^2(x)\right)}{\sin^2(x)} \\
&= \frac{-1}{\sin^2(x)} \text{ (Trig Identity)} \\
&= -\csc^2(x).
\end{aligned}$$

An alternative route that some students took was:

$$\begin{aligned}
\frac{d}{dx}(y) &= \frac{d}{dx}\left[\frac{\cos(x)}{\sin(x)}\right] \\
&= \frac{\frac{d}{dx}(\cos(x)) \cdot \sin(x) - (\cos(x))\frac{d}{dx}(\sin(x))}{\sin^2(x)} \text{ (Quotient Rule)} \\
&= \frac{-\sin(x)\sin(x) - \cos(x)\cos(x)}{\sin^2(x)} \\
&= -\frac{\sin^2(x)}{\sin^2(x)} - \frac{\cos^2(x)}{\sin^2(x)} \\
&= -1 - \cot^2(x).
\end{aligned}$$

Either of these will be accepted as correct.

Final Answer:

$$y'(x) = -\csc^2(x) = -1 - \cot^2(x).$$